1. Practice with logarithms. Simplify the following expressions.
   a. \(2^{\lg n}\)
   b. \(\lg(2^n)\)
   c. \(\lg(16 n^2)\)
   d. \(2 \lg(n^2)\)
   e. \(\frac{1}{4} 2^{n+2}\)
   f. \(\ln(16)/\ln(2)\)
   g. \(\lg(2 * 2 * 2 * 2 * 2 * 2 * 2 * 2)\)
   h. \(\ln(e * e * e * e * e * e * e * e)\)
   i. \(\log(100000)\)
   j. \(\lg(n^2) - \lg(n)\)

2. Prove from the definition that \(2n^2 + 100n \log n + 1000 = O(n^2)\)

3. For what values of \(n\) is \(50 n \lg n\) greater than \(0.5 n^2\)? Why do we say that \(0.5 n^2\) is asymptotically larger, if \(50 n \lg n\) is larger for many values? (Hint: you may need to graph the functions or play around with your calculator.)
Part B.

1. Suppose an algorithm A has runtime \( n^2 \) for some problems of size \( n \) and \( n^3 \) for others, when \( n \geq 100 \). When \( n < 100 \), the runtime is \( n^4 \). Circle all of the following that are known to be true about the runtime of A.

   **Worst case:**
   - \( O(n) \)
   - \( O(n^2) \)
   - \( O(n^3) \)
   - \( O(n^4) \)
   - \( \Omega(n) \)
   - \( \Omega(n^2) \)
   - \( \Omega(n^3) \)
   - \( \Omega(n^4) \)
   - \( \Theta(n) \)
   - \( \Theta(n^2) \)
   - \( \Theta(n^3) \)
   - \( \Theta(n^4) \)

   **Average case:**
   - \( O(n) \)
   - \( O(n^2) \)
   - \( O(n^3) \)
   - \( O(n^4) \)
   - \( \Omega(n) \)
   - \( \Omega(n^2) \)
   - \( \Omega(n^3) \)
   - \( \Omega(n^4) \)
   - \( \Theta(n) \)
   - \( \Theta(n^2) \)
   - \( \Theta(n^3) \)
   - \( \Theta(n^4) \)

2. True or false?
   - \( n^{1.001} = O((\log n)^{97}) \)
   - \( 2^{2n} = O(2^n) \)
   - \( 2^{n^7} = O(2^n) \)
   - \( n^{2.01} = O(10 n^3) \)

3. What is the asymptotic runtime of the following segments of code?

   1. for (i=1; i<n; i++)
      for (j=1; j<n; j++)
        cout << "Hello world!\n";
      for (i=1; i<n; i++)
        for (j=1; j<n; j++)
          cout << "Hello world!\n";

   2. for (i=1; i<n; i++)
      for (j=1; j<n; j*=2)
        for (k=1; k<n; k++)
          cout << "hello world!\n";

   3. for (i=n; i>1; i/=2)
     cout << "Hello world!\n";

   4. for (i=1; i<n; i *= 3)
     cout << "Hello world!\n";

4. Suppose your friend says that she is thinking of a positive integer (without telling you that it’s between two other numbers, for example). Explain how you could guess the number with \( O(\log n) \) guesses in which your friend replies “lower,” “correct,” or “higher,” where \( n \) is the number your friend is thinking of.