

The accumulation of roundoff errors in a chain of calculations may be so large that the result of the calculations is far from the correct value, which can lead to serious consequences. A real-world example of this is the Patriot missile failure described in Section 1.4. A Patriot missile battery in Dhahran, Saudi Arabia, failed to intercept a Scud missile during the Gulf War in 1991 with the result that 28 American soldiers were killed and 98 were injured. The cause of the failure was identified as an inaccurate calculation of the elapsed time since start-up due to roundoff errors of the kind we just described. Specifically, the time measured by the system clock was multiplied by 0.1 to produce the time in tenths of seconds. Like most decimal fractions, the binary representation of 0.1 does not terminate:

$$(0.000110011001100110011001100110011 \dots)_2$$

The 24-bit register in the Patriot stored only an approximation to this number,

$$(0.00011001100110011001100)_2$$

which is in error by approximately 0.000000095. The system had been running for 100 hours and during this time, the roundoff errors had accumulated to more than 0.34 seconds. During this time a Scud missile traveling approximately 5550 feet per second could travel about 1890 feet, which means that the calculated distance to the missile could be off by 1/3 mile.